ASG v1 EX 20.1 (variability - luminosity)

Since A is brighter, it must have a longer period of variability. By how much? First, how much larger is A's magnitude (m_A) than B's magnitude (m_B)? According to the modern scale of magnitude, the magnitude of an object goes down by 1 whenever the brightness goes up by a factor of $10^{0.4} \approx 2.512$. This is from the formula

$$m_A - m_B = -2.512 \log_{10}(I_A / I_B)$$

where I is the intensity (brightness) in watts/m^2. Therefore, the magnitude must increase by a factor of 5 to increase the brightness by a factor of $2.512^5 \approx 100$.

Now Shapely's formula relating magnitude and period is

$$m = -1.78 - 1.74 \log_{10}(T)$$ ← (a straight line if plotting m vs. $\log_{10}(T)$)

We see that to decrease m by 5, we must increase $\log_{10}(T)$ by $\frac{5}{1.74} \approx 2.87$. So $\log(T_A) - \log(T_B) = 2.87$.

Simplifying: $\log\left(\frac{T_A}{T_B}\right) = 2.87$ or $\frac{T_A}{T_B} = 10^{2.87} = 741$

Thus $\boxed{T_A \approx 741\, T_B}$