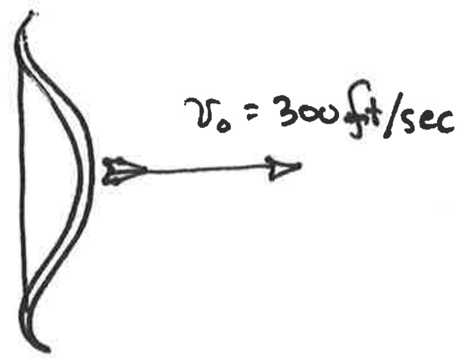
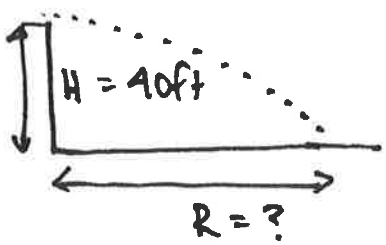
ASG v2 Ex 11.2 (Archery)



$g = 32$ ft/sec^2

a)

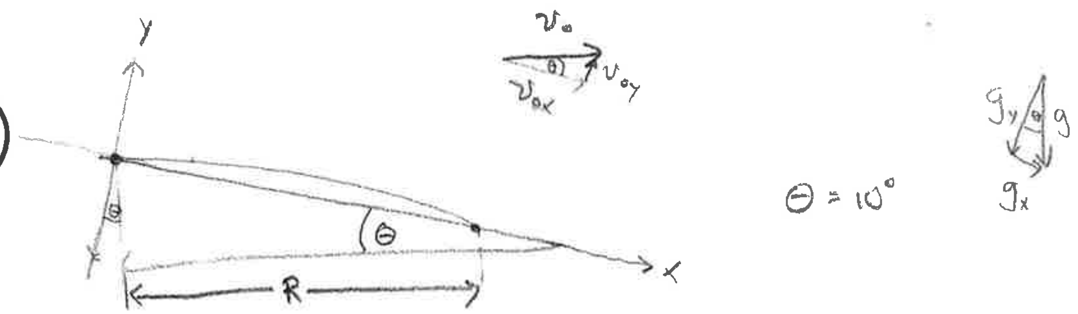


The time of flight is found by

$$H = \frac{1}{2} g t^2$$

$$t = \sqrt{2h/g} = 1.6 \text{ sec}$$

The range is found by $R = v_0 t = 470$ feet b.)



- If the archer fires downhill, what is the range?
- Set up a coordinate system whose abscissa is in the plane of the ground. Then the arrow experiences x-acceleration and y-acceleration. Its x-velocity is

$$v_x(t) = v_{0x} + g_x t \quad , \quad g_x = g\sin\theta \, , \quad v_{0x} = v_0\cos\theta$$

And its y-velocity is given by

$$v_y(t) = v_{0y} - g_y t \quad , \quad g_y = g\cos\theta \, , \quad v_{0y} = v_0\sin\theta$$

- The height of the arrow above the ground is given by

$$y(t) = v_{0y}t - \tfrac{1}{2} g_y t^2$$

which is zero at $t=0$ and at a later time given by

$$\tfrac{1}{2} g_y t_1^2 - v_{0y} t_1 = 0$$

$$t_1\left(\tfrac{1}{2} g_y t_1 - v_{0y}\right) = 0$$

$$t_1 = \frac{2v_{0y}}{g_y}$$

$$t_1 = \frac{2v_0}{g}\tan\theta = 3.3 \text{ seconds}$$

- In time t_1, the arrow will fly an x-distance

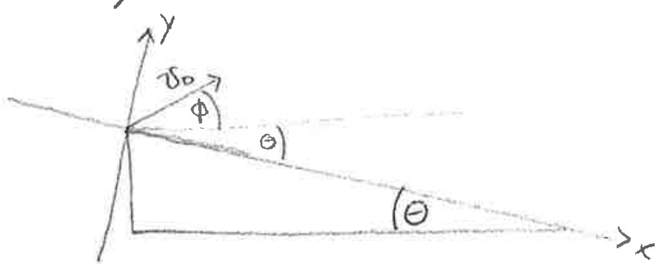
$$x(t_1) = v_{0x} t_1 + \frac{1}{2} g_x t_1^2$$

$$= v_0 \cos\theta \, t_1 + \frac{1}{2} g \sin\theta \, t_1^2 = 1000 \text{ feet}$$

- The range, R, is given by,

$$R = x \cos\theta = \boxed{990 \text{ feet}}$$

c). Now, how does the range depend on the firing angle, ϕ?



- Again, lets determine the x and y velocities:

$$\begin{cases} v_x(t) = v_{0x} + g_x t \\ v_{0x} = v_0 \cos(\theta + \phi) \\ g_x = g \sin\theta \end{cases}$$

$$\begin{cases} v_y(t) = v_{0y} + g_y t \\ v_{0y} = v_0 \sin(\theta + \phi) \\ g_y = g \cos\theta \end{cases}$$

- The height is given again by

$$y(t) = v_{0y} t - \frac{1}{2} g_y t^2$$

which has zeros at $t_0 = 0$ and

$$t_1 = \frac{2 v_{0y}}{g_y} = \frac{2 v_0 \sin(\theta + \phi)}{g \cos\theta}$$

- The x-coordinate is then

$$x(t_1) = v_{0x} t_1 + \frac{1}{2} g_x t_1^2$$

$$= v_0 \cos(\theta + \phi) t_1 + \frac{1}{2} g \sin\theta \, t_1^2$$

$$= \frac{v_0 \cos(\theta+\phi) \, 2 v_0 \sin(\theta+\phi)}{g \cos\theta}$$

$$+ \frac{1}{2} g \sin\theta \left(\frac{2 v_0 \sin(\theta+\phi)}{g \cos\theta} \right)^2$$

$$x(t_1) = \frac{2 v_0^2}{g} \left[\frac{\sin(\theta+\phi)\cos(\theta+\phi)}{\cos\theta} + \frac{\sin^2(\theta+\phi) \sin\theta}{\cos\theta^2} \right]$$

$$= \frac{2 v_0^2 \sin(\theta+\phi)}{g \cos\theta} \left[\cos(\theta+\phi) + \sin(\theta+\phi) \tan\theta \right]$$

- The rng is then

$$R = \frac{2 v_0^2}{g} \sin(\theta+\phi) \left[\cos(\theta+\phi) + \sin(\theta+\phi) \tan\theta \right]$$

- If I replace $\theta + \phi = \alpha$ (to simplify) then

$$R = \frac{2v_0^2}{g} \sin\alpha \left[\cos\alpha + \sin\alpha \tan\theta\right]$$ is the range

$$= \frac{2v_0^2}{g}\left[\frac{1}{2}\sin 2\alpha + \tan\theta \, \frac{1}{2}(1 - \cos 2\alpha)\right]$$

$$R = \frac{v_0^2}{g}\left[\sin 2\alpha + \tan\theta - \tan\theta\cos 2\alpha\right]$$

This has a max @ $\alpha = 50^\circ$

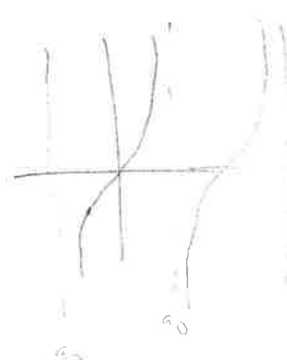
- For maximum range $\left.\frac{dR}{d\alpha}\right|_{\alpha_0} = 0$

$$\frac{dR}{d\alpha} = \frac{v_0^2}{g}\left[2\cos(2\alpha_0) + 2\tan(\theta)\sin(2\alpha_0)\right] = 0$$

$$\Rightarrow -\cos(2\alpha_0) = \sin(2\alpha_0)\tan(\theta)$$

$$\frac{-1}{\tan\theta} = \tan(2\alpha_0)$$

$$\boxed{\alpha_0 = \frac{1}{2}\arctan\left(-\frac{1}{\tan\theta}\right)}$$



- If $\theta = 10^\circ$, then $\alpha_0 = -40^\circ$

But this is the minimum of R.

The maximum is at $\alpha_0 = +50^\circ$

So $10 + \phi = 50 \Rightarrow \boxed{\phi = 40^\circ}$