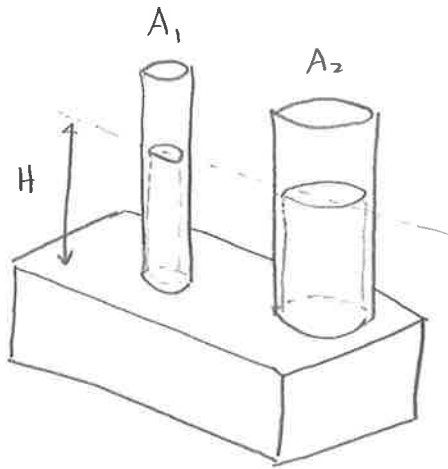


ASG 12

EX 14.2 (Fluid equilibrium & center of gravity)



$$A_1 = \pi \left(\frac{2\text{cm}}{2}\right)^2 = \pi \text{cm}^2$$

$$A_2 = \pi \left(\frac{4\text{cm}}{2}\right)^2 = 4\pi \text{cm}^2$$

$$H = 6\text{cm}$$

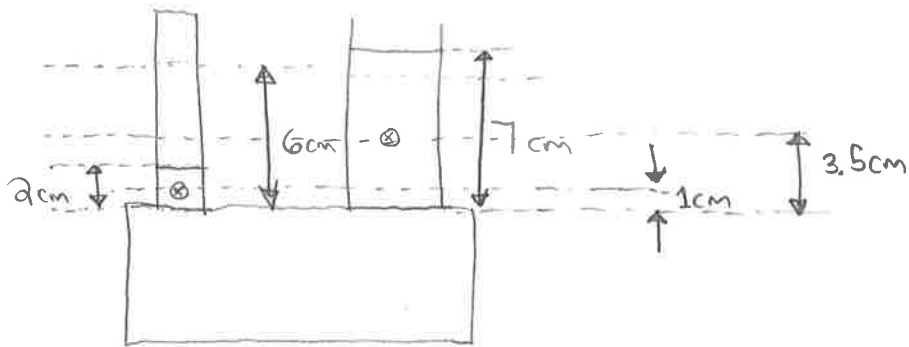
a.) The weights in the tubes are unequal.

Tube 1 carries $A_1 H \rho = 6\pi$ grams of water

Tube 2 carries $A_2 H \rho = 24\pi$ grams of water

They are in equilibrium because of Pascal's principle: the fluid in the vessel exerts an upward force equivalent to the weight of 6π grams of water on the left column and 24π grams of water on the right column, since the area of the right opening is 4 times as big as the area of the left opening.

b.) As shown, the center of gravity of the water in the columns is 3cm above the box. If the water in the left is depressed by 4cm, then 4π cc of water moves to the right column, and the water level in the right goes up by 1 cm, as shown here:



So what happens to the center of gravity of the fluid?
 The c.o.g. of the right column is now at 3.5 cm above the box, of the left column is now at 1 cm above the box (as shown by the symbols ⊗ above). So

$$y_{\text{cog}} = \frac{(1\text{cm})(2\pi \text{cc}) + (3.5\text{cm})(7 \times 4\pi \text{cc})}{[(2\pi) + (7 \times 4\pi)] \text{cc}} = 3.333\text{cm}$$

The c.o.g of the fluid has ascended by $\frac{1}{3}$ cm.

c.) More generally, if the left side is pushed down by distance δy_1 , the right goes up by $\frac{A_1}{A_2} \delta y_1 = \delta y_2$

$$\begin{aligned} \text{Since } y_{\text{cog}} &= \frac{\left(\frac{y_1}{2}\right)(w_1) + \left(\frac{y_2}{2}\right)(w_2)}{w_1 + w_2} \\ &= \frac{\left(\frac{y_0 + \delta y_1}{2}\right) \left((y_0 - \delta y_1)A_1\right) + \left(\frac{y_0 + \delta y_2}{2}\right) \left((y_0 + \delta y_2)A_2\right)}{(y_0 - \delta y_1)A_1 + (y_0 + \delta y_1)A_2} \end{aligned}$$

$$Y_{\text{cog}} = \frac{1}{2} \frac{(Y_0 - \delta y_1)^2 A_1 + (Y_0 + \frac{A_1}{A_2} \delta y_1)^2 A_2}{(Y_0 - \delta y_1) A_1 + (Y_0 + \frac{A_1}{A_2} \delta y_1) A_2}$$

Simplifying: take the initial height $Y_0 = 6$ and $\delta y_1 = \delta y$
and $A_2 = 4A_1$ and $A_1 = \pi$

$$Y_{\text{cog}} = \frac{1}{2} \frac{(6 - \delta y)^2 \pi + (6 + \frac{\delta y}{4})^2 4\pi}{(6 - \delta y)\pi + (6 + \frac{\delta y}{4})4\pi}$$

$$Y_{\text{cog}} = \frac{1}{2} \frac{(6 - \delta y)^2 + 4(6 + \frac{\delta y}{4})^2}{(6 - \delta y) + 4(6 + \frac{\delta y}{4})}$$

This is plotted below. Significantly, the C.O.G. is lowest

when δy (or x)

is zero. That is,

when the fluid

height is the

same in both

columns, as

Pascal mentions.

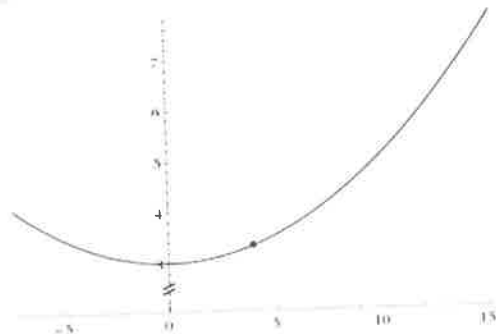
next calculation

$$\frac{(6-x)^2 + 4(6 + \frac{x}{4})^2}{2((6-x) + 4(6 + \frac{x}{4}))} \quad \text{where } x=4$$

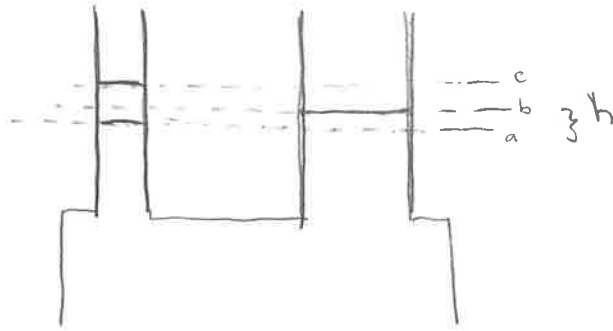
Result:

$$\frac{10}{3}$$

Plot



If 2cm of alcohol are poured atop the left column of water, it will descend until the pressure at the level of the bottom of the alcohol is equal on both sides.



That is, the pressure at level a is the same in the right and left columns.

$$\begin{aligned}
 P_{a, \text{left}} &= \frac{\text{Weight alcohol}}{\text{Area 1}} \\
 &= \frac{(2 \text{ cm})(\pi \text{ cm}^2)(\rho_{\text{alcohol}})}{\pi \text{ cm}^2} \\
 &= 2 \rho_{\text{alcohol}}
 \end{aligned}$$

$$\begin{aligned}
 P_{a, \text{right}} &= \frac{(h \text{ cm})(4\pi \text{ cm}^2)(\rho_{\text{water}})}{4\pi \text{ cm}^2} \\
 &= h \rho_{\text{water}}
 \end{aligned}$$

$$\text{So } h = \frac{2 \rho_{\text{alcohol}}}{\rho_{\text{water}}} = (2)(0.79) = 1.58 \text{ cm}$$

The water in the right column is 1.58 cm above the water level in the left column