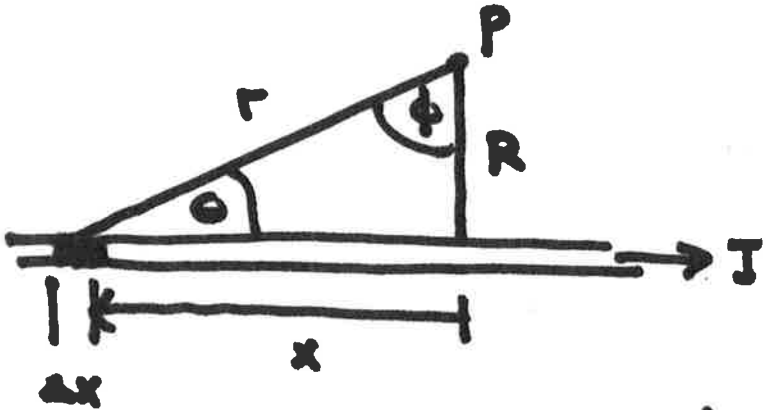
Ex 8.4 (Biot-Savart law)



We wish to find B at point P. We must sum the dB (small fields) caused by each segment of length dx. This amounts to integrating

$$B = \int dB = \int_{x=-\infty}^{x=\infty} \frac{\mu}{4\pi} \frac{I\,dx \sin\theta}{r^2}$$

using the Biot-Savart law (Eq. 8.3). x goes from $-\infty$ to $+\infty$. But θ is a function of x. So we must rewrite this integral in terms of one variable. Notice

$\sin\theta = \cos\phi$ (trigonometry)

$x = R\tan\phi$ (again: trig)

$dx = \dfrac{R\,d\phi}{\cos^2\phi}$ (calculus)

$$\frac{R}{r} = \cos\phi \quad \text{(trig again)}$$

$$\frac{1}{r^2} = \frac{\cos^2\phi}{R^2} \quad \text{(finally)}$$

- Now we can substitute these into our integral:

$$B = \int \frac{\mu}{4\pi} I \frac{\cancel{\cos^2}\phi}{R^2} \frac{R\,d\phi}{\cancel{\cos^2}\phi} \cos\phi$$

$$= \int_{\phi_1}^{\phi_2} \frac{\mu}{4\pi} \frac{I}{R} \cos\phi \, d\phi$$

$$= \frac{\mu I}{4\pi R} \int_{\phi = -\pi/2}^{\phi = \pi/2} \cos\phi \, d\phi$$

$$= \frac{\mu I}{4\pi R} \left[\sin\phi \right]_{-\pi/2}^{\pi/2}$$

$$B = \frac{\mu I}{2\pi R} \quad \text{(as expected)}$$